

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the even and odd components of the signal $x(n) = \{1, 0, -1, 2, 3\}$. (8)

- (ii) Find the fundamental period of the signal $x(t) = e^{j\frac{7\pi}{3}n}$. (8)

Or

- (b) (i) Check the system $y(n) = \log_{10}|x(n)|$ is linear, time invariant, causal and static. (10)

- (ii) Find the summation $\sum_{n=0}^5 \delta(n+1)2^n$. (6)

12. (a) (i) Prove the scaling and time shifting properties of Laplace transform. (8)

- (ii) Determine the Laplace transform of $x(t) = e^{-at} \cos at u(t)$. (8)

Or

- (b) (i) State and prove the Fourier transform of the following signal in terms of $X(j\omega)$; $x(t - t_0)$, $x(t)e^{jat}$. (8)

- (ii) Find the complex exponential Fourier series coefficient of the signal $x(t) = \sin 3\pi t + 2\cos 4\pi t$. (8)

13. (a) (i) Determine the impulse response $h(t)$ of the system given by the differential equation $\frac{d^2 y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$ with all initial conditions to be zero. (8)

- (ii) Obtain the direct form I realization of $\frac{d^2 y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4y(t) = \frac{dx(t)}{dt}$. (8)

Or

- (b) The system produces the output $y(t) = e^{-t}u(t)$ for an input $x(t) = e^{-2t}u(t)$. Determine

- (i) frequency response
 (ii) magnitude and phase of the response
 (iii) the impulse response. (16)

14. (a) (i) Determine the discrete time Fourier transform of $x(n) = a^{|n|}$, $|a| < 1$. (8)

(ii) Find the z transform and ROC of the sequence $x(n) = r^n \cos(n\theta)u(n)$. (8)

Or

(b) (i) State and prove the following properties of z transform

(1) Linearity

(2) Time shifting

(3) Differentiation

(4) Correlation. (8)

(ii) Find the inverse z-transform of the function

$$X(z) = \frac{1+z^{-1}}{\left(1-\frac{2}{3}z^{-1}\right)^2} \text{ROC } |z| > \frac{2}{3} \quad (8)$$

15. (a) (i) Find the system function and the impulse response $h(n)$ for a system described by the following input-output relationship

$$y(n) = \frac{1}{3}y(n-1) + 3x(n). \quad (6)$$

(ii) A linear time-invariant system is characterized by the system function

$$H(z) = \frac{3-4z^{-1}}{1-3.5z^{-1}+1.5z^{-2}}$$

Specify the ROC of $H(z)$ and determine $h(n)$ for the following conditions

(1) The system is stable

(2) The system is causal

(3) The system is anti-causal. (10)

Or

(b) (i) Derive the necessary and sufficient condition for BIBO stability of an LSI system. (6)

(ii) Draw the direct form, cascade form and parallel form block diagrams of the following system function : (10)

$$H(z) = \frac{1}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

